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Neutrino masses and mixings: What do they mean?

Or: 'Going beyond the Standard Model: but where?'

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Abstract. Neutrino masses show that the standard model of the elementary particles and interactions is incomplete and suggest the existence of physics at high mass scales. Furthermore, the difference between quark and lepton mixing raising new questions. We discuss some theoretical attempts to address these questions emphasizing their limitations and the potential.

PACS. 14.60.Pq Neutrino mass and mixing – 12.10.-g Unified field theories and models

Theoretical speculations like this one had always limited space at the HEP conferences. But even accounting for that, I am surprised that the sessions on *Neutrinos* or *Beyond the Standard Model* devoted very little space to discuss how to use neutrinos as keys to the physics beyond the standard model. This is understandable since in the past, most popular models ideas or prejudices on neutrinos failed and led us on what we know now to be wrong tracks. But going from blind beliefs to total skepticism, or abandoning the hopes to find a theory of fermion masses because of previous failures, does not seem to me to be the right attitude.[1]

This work is organized in 3 parts. Sec.1, titled 'Neutrino Masses and Mixings', draws the context of the discussion; Sec.2, titled 'What Do They Mean', is devoted to discuss some ideas that are in our view interesting. The last section is a brief and necessarily provisional conclusion. (I thank here all my collaborators and in particular B. Bajc, V. Berezinsky, G. Senjanović, A. Strumia and don't be worried, as usual, mistakes in the writeup are mine.)

1 Neutrino masses and mixings ...

1.1 Standard Model, Annum Domini 2003

By definition, the standard model of elementary particles (SM) is based on the following principles:

a) It is a renormalizable theory
b) Its gauge group is $\mathrm{SU}(3)_c\times\ \mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$
c) The fermions are $q = \begin{pmatrix} u \\ d \end{pmatrix}$, $\ell_e = \begin{pmatrix} \nu_e \\ e \end{pmatrix}$, $\begin{matrix} u^c \\ d^c \end{matrix}$, $\begin{matrix} \bullet \\ e^c \end{matrix}$ (quarks and leptons, matter fields) come in 3 families.
d) Symmetry breaking and fermion mass generation arise due to vacuum expectation value of the higgs field $H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$, namely $\langle H^0 \rangle = 174$ GeV.

As such, this theory has a number of implications:
⇒ *The baryon and 3 lepton numbers B, L_e, L_μ, L_τ, are accidental (global) symmetries* (recall, the differences as B-L have no anomalies)
⇒ *Neutrinos are massless* (as we all know, this prediction is contradicted by a large number of experiments)
⇒ *The higgs sector is untested and 'troublesome'* (supersymmetry 'solves' the problem of mass hierarchy, but introduces new parameters and questions. E.g., B and L are not accidental symmetries in the minimal supersymmetric extension[2])
⇒ *There are a lot of free parameters* (recall, Grand Unification Theories (GUT) help on gauge couplings, the problem is Yukawa couplings (and Higgs sector). Large classes of GUT explain why neutrino masses are small–'seesaw'.

1.2 Neutrino masses as the most urgent need of SM

No doubt: we have to discuss how to modify the standard model to account for massive neutrinos. There are several simple and reasonable possibilities:

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[1] Disclaimer: I accepted an assignment from the Conveners so I will do my best in honoring their request. However I will not 'defend theory' but rather I will attempt a comparison of various views and ideas, emphasizing their limitations and weaknesses. Furthermore, I will propose more questions than answers, I will stress the role of assumptions, I will be not complete and ... I will be frankly biased.

[2] But to be fair, we have to admit that–even assuming supersymmetry–the question about the cosmological constant remains unsolved.

1. Relax the hypothesis of *renormalisability*:
$$\mathcal{L} \ni \frac{1}{M_x}(H\ell)^2 \Rightarrow m_\nu = \frac{\langle H^0\rangle^2}{M_x}$$
2. Enlarge the *matter* content
$$\mathcal{L} \ni yH\ell\nu^c \Rightarrow m_\nu = y\langle H^0\rangle \ \text{(Dirac mass)}$$
or, noting that ν^c is a SM singlet
$$\mathcal{L} \ni yH\ell\nu^c + \frac{M_\nu}{2}\nu^c\nu^c \Rightarrow m_\nu = \frac{y^2\langle H^0\rangle^2}{M_\nu}$$
3. Enlarge the *Higgs* content
$$\mathcal{L} \ni [f(\ell\sigma\ell) + \mu(H^c\sigma H^c)]\mathbf{T} \Rightarrow m_\nu = \frac{f\langle H^0\rangle^2}{M_T^2/\mu}$$
4. ... and many more possibilities

(1=Effective, dim5 term; 2=Dirac mass and canonical or type I seesaw; 3=non-canonical or type II seesaw; 4=e.g., loop induced mass. See e.g., the book [1]). The Dirac case requires that the Yukawa couplings are very small. The other cases instead relates the observed smallness of neutrino masses to new physics at higher scale, still to be discovered. A big question is: *Can we distinguish among them?* Let us discuss it shortly:

• If we have Dirac neutrinos, no ΔL_e effect should be seen (though the converse is not true)

• If we have a model with few parameters, it can lead to testable predictions, perhaps relating neutrino parameters among them.

• But most often, to test a model one needs also other observables (e.g., masses of charged fermions).

In our view, the Dirac case amounts to assume that there is nothing to explain about neutrino masses; said otherwise, if the reader believes in Dirac neutrinos, the paper ends here. But even barring Dirac neutrinos, it should be clear that there are (too) many theoretical options. How to decide? Certainly, it is desirable that a model is "testable", but we should not *force* models to make predictions. Rather, we should try to see whether motivated theoretical setups lead to predictions–or they don't.

1.3 A pure experimental approach?

Here, we would like to discuss the new observable parameters. Let us stick to the case of a Majorana mass, that interestingly points to new physics at a higher scale. There are 9 (or more in general, n^2) parameters. The effective lagrangian is:

$$\mathcal{L} = \frac{1}{2}\ \mathbf{M}_{\ell\ell'}\ \nu_\ell^t\ C^{-1}\ P_L\ \nu_{\ell'} + h.c. \qquad \ell,\ell' = e,\mu,\tau,s,...$$

It is useful to decompose the mass matrix as follows:

$$\mathbf{M}_{\ell\ell'} = \sum_{j=1}^{n} U_{\ell j}^* \cdot m_j \cdot \mathrm{e}^{i\xi_j} \cdot U_{\ell' j}^*$$

(my phase convention is $U_{ej} \in I\!R$: in this way, the oscillation phase ϕ is not relevant for $0\nu 2\beta$). In principle, we could measure (see e.g., [2]):

- 3 (or $\frac{n(n-1)}{2}$) angles θ_{ij} and 1 (or $\frac{(n-1)(n-2)}{2}$) phase ϕ from oscillations
- 2 (or $n-1$) $\Delta m_{ij}^2 = m_i^2 - m_j^2$'s from oscillations
- $|\mathbf{M}_{ee}| = |U_{ej}^2 m_j e^{i\xi_j}|$ from neutrinoless double beta decay (up to nuclear uncertainties)
- $|U_{ej}^2| m_j^2$ (more parameters??) from β decay

Today we know 4 of these, and we are not certain that the number of light neutrinos is $n=3$. In future, 1 or 2 new parameters could be measured. Even assuming the simpler three neutrino picture we see that experimental investigations have a limited reach since 9 is larger than 4 (or 6). Thus, theoretical considerations are needed or at least welcome. We feel that a list of pressing questions should include:

- Why the mass hierarchy $(\Delta m_{sol}^2/\Delta m_{atm}^2)^{1/2} \sim 1/6 - 1/5$ is so weak?
- Why the leptonic mixing angles are so large (when those of the quarks are so small)
- It θ_{23} very close to maximal? Is θ_{13} very small? (or there are deviations close to the presently allowed ones, $\sim 1/6$ radians)?

As a first attempt, we could try to do as little as we can, hoping to avoid prejudices and mistakes. Suppose there are 3 neutrinos that mix among them. Suppose their spectrum is hierarchical and does not have main degeneracies. Then, the largest mass scale is $m = (\Delta m_{atm}^2)^{1/2} \sim$ 45 meV, and the mass matrix at leading order is

$$\mathbf{M} \approx \frac{m}{2}\begin{pmatrix} 0\ 0\ 0 \\ 0\ 1\ 1 \\ 0\ 1\ 1 \end{pmatrix} \qquad (1)$$

It is different from what many of us expected, but it is already something. Note that there is nothing like third family dominance, rather, the second and third families have the same importance in the mass matrix–there is a 'dominant block'. However, its determinant is not of the order of $\mathbf{M}_{\mu\mu}, \mathbf{M}_{\mu\tau}$ or $\mathbf{M}_{\tau\tau}$ but smaller (it arises about at the same order of the zeroes of the matrix given above).[3]

1.4 Summary, ideology, and plan

Let us conclude this section, by offering a summary, exposing the underlying ideology and describing the plan of the rest of the work.

⋆ If the SM is not thought as a fundamental theory, neutrino masses, proton decay and possibly other observable

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[3] A remark on this mass matrix (first introduced by Berezhiani and Rossi [3]): very often, this is taken as starting point in theoretical works, but it is not always clear if there are deeper reasons, than the simple considerations exposed here.

phenomena exist due to new physics at higher mass scales [4]. In this context, one understands why neutrinos are light [5].
$\star$ 1970 and 1980 were the years of GUT and supersymmetry. Both theories (principles) have roots in the SM; the 1^{st}, a natural extension of its principles (in particular, gauge principle), the 2^{nd} of its scalar sector. Neutrino masses lead to a renewed interest in GUT, as SO(10) [6]. This fact does not weaken the motivations for supersymmetry.
$\star$ The structure of fermion masses and mixing (neutrinos included) *seems* unexplained. Also, quark and lepton masses seem not unified. Since 70's, this consideration led many people (e.g. [7]) to suspect that there is some 'physics of flavor' to be discovered.
$\star$ It is important to update these considerations, explore their potential and understand their limitations. We begin with neutrino mass scales (including Planck scale physics and R-parity violating supersymmetry), continue with flavor symmetries and conclude with SO(10).

In short: neutrino masses are a window beyond SM, but don't forget the rest (in particular, fermion masses)

2 ... What do they mean?

I will discuss now a number of theoretical hypotheses, proposals, theories, guesses, models (the reader please decide what is what) doing my best to emphasize their potential and limitations. As discussed above, I will always stick to the assumption that the smallness of neutrino masses is due to some new physics at a very high energy scale.

2.1 The scales of new physics

Here, we would like simply to write down explicitly which are the expected scales of new physics.

Of course, we begin with neutrino masses [8]. Making reference to previous formulae, we get the following estimations for the mass of the super-heavy right handed neutrinos, or of the triplet:

$$\sqrt{\Delta m^2_{atm}} = \xi \frac{y^2 \langle H^0\rangle^2}{M_R} \quad \Rightarrow \quad M_R \sim 1 \cdot 10^{14} y^2 \text{ GeV}$$

$$\sqrt{\Delta m^2_{atm}} = \xi f \frac{\mu \langle H^0\rangle^2}{M_T^2} \quad \Rightarrow \quad M_T \sim 4 \cdot 10^{15} f \sqrt{\frac{M_{GUT}}{\mu}} \text{ GeV}$$

(ξ is a renormalization group factor). It should be noted, that these big mass scales re-propose the hierarchy problem of standard model [9].

Next let us consider what is suggested by gauge coupling unification [10]:

$$\text{Supersymmetric GUT} \quad \Rightarrow \quad M_{GUT} \sim 2 \cdot 10^{16} \text{ GeV}$$

This assumes the existence of the so called 'great desert' above the scale of supersymmetry. The latter in turn should be close to the electroweak scale. (One can note that this scale is not very much different from those suggested by neutrino masses.)

Finally, we have a scale coming from a very popular mechanism for baryogenesis [11]:

$$\text{Leptogenesis} \quad \Rightarrow \quad M_{leptog.} \sim 10^{10} - 10^{12} \text{ GeV}$$

However, this result is correct if the dominant contribution comes from the right-handed neutrinos and this is not necessarily the case: other particles can contribute to the process of leptogenesis, in particular, the same triplet mentioned here. In this case, the mass scale can increase and become closer to the scales previously considered [12].

2.2 An example of a scale 'too big'

It has been argued that *quantum gravity effects* could give a contribution to neutrino masses:

$$\mathbf{M}_\nu = \frac{\langle H^0\rangle^2}{M_{Planck}} \begin{pmatrix} 1\,1\,1 \\ 1\,1\,1 \\ 1\,1\,1 \end{pmatrix}$$

Definitively, we cannot explain the observed oscillations by this mass matrix: $M_{Planck} = 1.2 \times 10^{19}$ GeV is too large.

Still, a contribution to the neutrino mass matrix from Planck scale effects can be of some interest [13]:
$\star$ It yields a *lower limit on* U_{e3} (together with other small effects like renormalization).
$\star$ In model with mirror matter, it leads to *new oscillations* on very long scales of distance. (More in detail, this could affect solar, supernova, ultra-high-energy neutrinos). This is an example of (theoretically motivated) sterile neutrinos, important only for astrophysics or cosmology.

2.3 An example of a scale 'too small'

The higgs $H_d = (H^0, H^-)$ and lepton $\ell = (\nu_e, e)$ have the same quantum number, but the first is a boson, the second is a fermion. In supersymmetric standard model bosons and fermions unify and this allows terms like

$$\mathbf{L}H_u,\ \mathbf{L}LE^c,\ \mathbf{L}QD^c$$

This means that the accidental symmetries of SM are lost in supersymmetric context.

Seeing that optimistically, we have a tool to generate neutrino masses, e.g., mixing the neutrinos with the neutralinos χ (that play the role of right handed neutrinos). A good cold dark matter candidate is lost, but $BR(\chi \to W^\pm \mu^\mp) \sim BR(\chi \to W^\pm \tau^\mp)$ could be seen at colliders [14].

But the reference mass scale is $M_\chi \sim$ TeV, by far too low: *How to avoid big ν masses naturally?* (not to mention the troubles with proton decay, when we include in the superpotential also the term $\mathbf{U^c D^c D^c}$). I have no good answers, so I will stop the discussion here.

2.4 Flavor symmetries

Let us assume that the basic structure of neutrino mass matrix reflects a U(1)$_e$ (electronic) selection rule [15,16]:

$$\mathbf{M}_\nu \overset{\mathcal{O}(1)}{=} \frac{\langle H\rangle^2}{2M_x} \begin{pmatrix} \varepsilon^2 & \varepsilon & \varepsilon \\ \varepsilon & 1 & 1 \\ \varepsilon & 1 & 1 \end{pmatrix} \qquad (2)$$

Note the unspelled $\mathcal{O}(1)$ coefficients, which are to be seen as an assessment of what we ignore (in other words, this is a *class* of mass matrices). Some remarks are in order:

- The mass scale where M_x is fixed by hand (with some seesaw in mind); but the *adimentional quantities* can be predicted.
- Certain *special values of* ε as θ_C or $(m_\mu/m_\tau)^{1/2}$ are motivated by quark or charged leptons masses.
- This $\mathbf{M}_\nu$ is compatible with *seesaw* of type I or II (triplet). Lepton mixing could come from neutral or charged fermions (or generally from both).

Let us get first a qualitative understanding of how this ansatz works. We will follow three steps:
1) It is evident tht $U_{\mu 3}$ and $U_{\tau 3}$ (mixings of ν_μ and ν_τ) are large as we want; U_{e3} suppressed at the desired level by ε. In other terms, all is fine with atmospheric neutrinos.
2) The problem is with solar neutrinos, since typically $\mathcal{O}(1)$ coefficients yield two 'large' eigenvalues. We are facing the question [16]: *Why $\mu\tau$ block of* $\mathbf{M}_\nu$ *has a small determinant?* Suppose we overcome this stricture somehow. In the rotated basis, the mass matrix will looks as follows:

$$\mathbf{M}_\nu \to \frac{m}{2} \begin{pmatrix} \varepsilon^2 & \varepsilon & 0 \\ \varepsilon & 1/3 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

Thus, to have a large θ_{12}, ε should be not too small.
3) In this manner, it seems we can get all we want

- a large *usually not maximal* atmospheric mixing θ_{23};
- $\theta_{13} \sim \varepsilon$ not too small–*fine for experiments*,
- solar neutrino oscillations,
- $(\mathbf{M}_\nu)_{ee} = (\Delta m^2_{atm})^{1/2}\varepsilon^2$–*not that fine for exp.s.*

Now let us quantify whether this ansatz works well enough [17]. We can take $\mathcal{O}(1)$ coefficients with random phase and modulus, say, =1, =1 ± 20 %, =1 ± 100 % and assess predictions on a statistical basis. Taking as successful a model that reproduces the data within 2σ, there is no problem to satisfy each *single* experimental cut, in the sense that this happens with reasonabe chance probability. But these cuts should be satisfied *together*. When this condition is enforced, succeess percentages become rather small, about 0.1 %. Thus, the data are telling us much more than this ansatz is able to predict. In other words, such an ansatz is good only as a first approximation.

The message is: in order to progress, we need a theory of $O(1)$ coefficients (something more, or something else).

2.5 A Predictive supersymmetric GUT

Let us start from the question: What is a minimal SO(10) model *for fermion masses*? Consider the fermion bilinears:

$$\mathbf{16_M 16_M = 10 + 120 + 126}$$

Thus, we see which Yukawa coupling we can have. A $\overline{\mathbf{126}}_\mathbf{H}$-plet is needed to decouple ν_R. We also take a $\mathbf{10_H}$-plet. that does well with third family. In this way, we have a lot of ways to break SU(2). Let us show this using the familiar SU(5) language: (a) $\overline{\mathbf{126}}_\mathbf{H}$ contains 1_H, 5_H, 45_H and 15_H ($\ni$ 1 SU(2) singlet, 2 doublets, 1 triplet); (b) $\mathbf{10_H}$ decomposes into $5_H + \overline{5}_H$ ($\ni$ 2 SU(2) doublets). When accounting for fermion masses, a natural possibility is to take advantage of all these Higgs fields [18,19]. Fermion masses are constrained

$$\begin{array}{ll} Y_u = \cos\theta_u Y_{10} + \sin\theta_u Y_{126} & u \text{ quarks} \\ Y_D = \cos\theta_u Y_{10} - 3\sin\theta_u Y_{126} & \nu \text{ (Dirac)} \\ Y_d = \cos\theta_d Y_{10} + \sin\theta_d Y_{126} & d \text{ quarks} \\ Y_e = \cos\theta_d Y_{10} - 3\sin\theta_d Y_{126} & \text{charg. leptons} \\ Y_{\nu_L\nu_L} = Y_{\nu_R\nu_R} = Y_{126} & \text{other } \nu \text{ coupl.} \end{array}$$

but apparently they do not come out wrong [20,21] (there are also a few phases, here negleted). In a sense, we can ask this minimal SO(10) model to provide us with a theory of $\mathcal{O}(1)$ coefficients. Note in particular when the SU(2) doublets contained in the $\overline{\mathbf{126}}$-plet gets a VEV, we depart from the SU(5) prediction $Y_d = Y_e$.

We will discuss now the potential of this model to explain neutrino masses. Let us start from an apparently naive statement, that their masses is different from those of charged fermions because there is a special mass mechanism. The fact is that we have a special mass mechanism at our disposal, that is the non-canonical seesaw $\mathbf{M}_\nu \propto Y_{126}$ that obtains if the triplet gets a VEV. This position is consistent with 2^{nd} and 3^{rd} family charged fermion masses. Furthermore,

$$\begin{cases} \mathbf{M}_e = vY_{10} - 3v'Y_{126} \\ \mathbf{M}_d = vY_{10} + v'Y_{126} \end{cases} \Rightarrow \mathbf{M}_\nu \propto \begin{pmatrix} 0 & 0 \\ 0 & m_b - m_\tau \end{pmatrix} \qquad (3)$$

Thus, large mixing needs $b-\tau$ *unification* at GUT scale; and further, $m_2/m_3 = 1/3 - 1/10$ as needed for LMA [19]. Mohapatra, Goh, Ng [22] found that the solar neutrino mixing does not come out wrong after including 1^{st} family. Also canonical seesaw was shown to work [21]. Both three flavor analyses [21] [22] find that θ_{13} should lie around ≈ 0.16. A good question is: *how solid is this prediction?* I don't know the answer.

Can we promote the model discussed till here (with $\mathbf{16_M}, \mathbf{10_H}, \overline{\mathbf{126}}_\mathbf{H}$ only) into a real theory? There are two serious problems (1) we cannot break the SO(10) symmetry fully (since the only SM singlet in $\overline{\mathbf{126}}_\mathbf{H}$ is an SU(5) scalar), and (2) we have problems with supersymmetry (D-term flatness cannot be arranged). We can fix them:
⋆ *To solve the latter problem, we add a* $\mathbf{126_H}$.
⋆ *To solve the first one, we add a* $\mathbf{210_H}$*, that can develop a VEV in the directions 24 and/or 75* [23,18,24,25,26].
A unique feature of **210** is that the SU(2) doublets and triplets also get a VEV, e.g., due to

$$\mathbf{10_H}\ \overline{\mathbf{126}}_\mathbf{H}\ \langle\mathbf{210_H}\rangle \to 5_H\ \overline{45}_H\ \langle 24_H\rangle$$

Thus we buy a model for symmetry breaking and we get for free the model for fermion masses described above.[4] At this point, another good question is: *what are the predictions for proton decay, leptogenesis, gauge coupling unification and lepton flavor violation?* Again, I do not know the full answer yet, but I wish you agree with me that this should be studied in detail [27].

3 Discussion

I did my best to be fair to Conveners assignment, that is to discuss *Neutrino Masses and Mixings: What Do They Mean?* Now, I must repeat that I am not sure that it is really possible to approach such a big question in a completely fair way. If I try, I have to admit that I mostly described approaches that I find promising–but that I have only a vague idea of the correct answer.

More in general, one has to admit that there are many open questions, already from the beginning: LSND? Large common mass for neutrinos? Inverted spectrum? Some physics beyond neutrino masses? Sterile neutrinos? (We would feel to define the simplest, or reference case by barring all these possibilities). And even worser, when trying to follow some theoretical speculation: Supersymmetry? Grand unification? Perturbativity? (Incidentally, we assumed all these boldly in Sec.2.5).

However, I believe that as a first step we should try to define the question, or in other words, we should choose a way to approach the problem and follow it till the end. And if we take this point of view ... maybe, we don't even need to *invoke* a sort of flavor symmetry in order to have a theory of fermion masses ... maybe, old good QFT is all we need to account for neutrino masses ... maybe, we are on the verge of gaining new views on old issues along the way–e.g., gauge coupling unification and proton decay ...

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[4] In this context, one appreciates the SM better. The same Higgs gives mass to the vector bosons and to the fermions: A miracle of economy. In comparison, this SO(10) models is much worser, since we need the **210** **126** and $\overline{\mathbf{126}}$ Higgs fields to break the gauge symmetry and the **10** and $\overline{\mathbf{126}}$ to generate fermion masses. However, the number of parameters of this supersymmetric SO(10) model is the same as the minimal supersymmetric SM with neutrino masses put by hand [26].